A Reasonable Doubt

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For a while, I have felt that there should be a greater emphasis on probability in mathematics education. All of us need to understand probability to better understand what is going on around us and make sound decisions. Here is an example:

Last Sunday, there was a cover story in the *New York Times Magazine* on the havoc wreaked by false positives from an inexpensive drug test. Applying some elementary probability, this outcome was completely predictable. In fact, this is an excellent example how important an understanding of probability is to public life. As I will show, the problem is not with the drug test, but with prosecutors' use of the results.

Let's look at a simple example: Suppose the test is pretty good:

- When the suspect is not using drugs, it tests negative 99% of the time. So, the false positive rate is 1%.
- When the suspect is actually on drugs, it tests positive for 99% of the time. So, the false negative rate is also 1%.

Now let's also suppose that 1% of suspects stopped by the police are, in fact, users.

Finally, let's suppose 10,000 people are stopped and tested by the various police forces. Under these very reasonable assumptions:

- The number of drug users in the 10,000 comes to 100 people. Of these 100 users. 99 will test positive.
- The remaining 9,900 are not drug users, and 1% of these will falsely test positive. This also comes to 99 folks.

As you see, of the 10,000 people tested, there will be 198 positive tests:

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- 99 true positives are guilty
- 99 false positives are innocent

So, if someone tests positive, there is a 50% chance they are guilty, and a 50% chance they are innocent.

The key point is:

If there is a test that can determine the guilt or innocence of a suspect, testing innocence is good evidence of being innocent, while testing guilty is ***not*** good evidence of guilt.

In our example, if the suspect is driving under the influence of a drug, then they will test positive 99% of the time. However, if the suspect tests positive, then he or she is actually driving under the influence 50% of the time.

The standard of guilt in criminal cases is "reasonable doubt" (http://legal-dictionary.thefreedictionary.com/reasonable+doubt). Clearly, 50% likelihood of innocence is reasonable doubt.

This is an example of straightforward application of the 300-year-old Bayes Theorem. It is troubling that such a well-understood phenomenon is still an issue in 2016. In fact, this is what is called by the Bayes community "the prosecutor's fallacy" (see https://en.wikipedia.org/wiki/Prosecutor%27s_fallacy).

The difference between the odds of testing positive if guilty and the odds of *being*guilty if testing positive is not intuitive. It takes study and practice. Hence, as I said, it should be part of everyone's education.

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